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AUTHOR	Suydam, Marilyn N.
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ABSTRACT

"Elmath," a computer-assisted instruction (CAI) course intended for in-service training of teachers of elementary school mathematics, has been translated into Spanish, together with accompanying handbook, achievement tests, and attitude scale. The English version has been used successfully by the Appalachia Regional Laboratory and by The Pennsylvania State University. Procedures followed in the translation of the course were these: (1) development of guidelines, (2) translation of the program, (3) exploration of potential sites for use of the course, and (4) preparation for computer input. While the course was coded for input into the IBM 1500 system, the text could be taken directly and coded for presentation on other equipment. The course has been recommended for use in Chile; this awaits further action. No other decision has been reached as to its future use. References are provided. The appendices contain: an outline of the content of "Elmath" by chapter, an evaluation of "Elmath" by Chilean educators, and a sample chapter of the Spanish translation of the course. (MF)

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Project No. 0-B-008
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DEVELOPMENT OF A COMPUTER ASSISTED INSTRUCTION COURSE FOR THE TRAINING OF SPANISH-SPEAKING TEACHERS IN ELEMENTARY SCHOOL MATHEMATICS

Marilyn N. Suydam

The Pennsylvania State University
University Park, Pennsylvania 16802

November 30, 1970

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PREFACE

The product that resulted from this project -- the Spanish translation of a mathematics program for elementary school teachers -- is the result of the efforts of many:

Dr. Joseph Alessandro, who not only supervised the work of students, but also spent some time in Latin America discussing potential use of the materials;

Maria Martinez, who with pleasant efficiency translated and reviewed huge portions of the material;

Elidia de Dolande, Delia Ortiez, Evangeline Negron Chugh, Carolina Acosta-Gonzalez who translated and reviewed other huge portions;

Beverly Brooks and Donna Beck, who typed cheerfully on the endless pages until they could happily file the completed materials.

All are to be commended for a task well done!

ABSTRACT/SUMMARY

DEVELOPMENT OF A COMPUTER ASSISTED INSTRUCTION COURSE
FOR THE TRAINING OF SPANISH-SPEAKING TEACHERS
IN ELEMENTARY SCHOOL MATHEMATICS
Grant No. OEG-2-700016(509)

The thirteen-chapter computer assisted instruction course elmath, which presents content and methods of teaching elementary school mathematics, has been translated into Spanish, together with accompanying Handbook, achievement tests, and attitude scale. It is now ready for input into a computer system.

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I. INTRODUCTION

A. Purpose of the Project

A computer-assisted instruction (CAI) course on the content and teaching of elementary school mathematics* has been translated into Spanish, and conceptually adapted for eventual presentation to Spanish-speaking teachers.

The project was based on three primary tenets:

(1) Spanish-speaking elementary school teachers need to improve their background on the content and teaching of elementary school mathematics as much as English-speaking teachers do.

(2) Computer-assisted instruction is a feasible means of presenting this content.

(3) The development of materials phase should precede any input of materials into the computer.

B. Rationale and Objectives

Researchers in elementary school mathematics have found that, while the level of mathematical competency of elementary teachers is improving, many teachers still do not have an adequate background. Griffin (1967) surveyed over 1000 teachers and found that they understood only half of the total topics on which they were tested and only a third of the modern topics. Williams (1966) also reported low levels of achievement when compared with pupils, while Stoneking and Welch (1961) reported that the amount of preparation was reflected in higher achievement scores. Kipps (1968) cited details of what teachers understood, and corroborated the need for improvement. Teachers were found to be weakest in whole number, decimal, and percentage concepts (Kenney, 1965). Few processes, concepts, and relationships were understood by the majority of teachers. Sparks (1961) and Riedesel (1964) found that progress in up-dating the mathematics of elementary school teachers has been slow. Various

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* This course, elmath, was developed at The Pennsylvania State University by C. Alan Riedesel, Marilyn N. Suydam, and Cecil R. Trueblood. Course development and course revision were conducted under grants from the U.S. Office of Education.

surveys by state departments of education (such as those reported from Colorado by Fisher, 1965), provide additional evidence on the current status of in-service teachers.

Research evidence indicates that a comparable situation exists among Spanish-speaking teachers (Alessandro, 1969). In fact, the average amount of mathematics background for Spanish-speaking teachers is well below that of English-speaking teachers from the same areas. Obviously, the children in the classes of these teachers need a sound mathematical background as much as other children do.

Thus, teachers lack basic mathematical concepts necessary in teaching the subject, and do not always use teaching strategies and procedures which have been shown to be particularly effective. That these teachers need to extent their knowledge and change their patterns of teaching to meet the challenge of current curriculum development and the rapid expansion of knowledge seems evident. In-service education provides one answer to meeting this need.

Recent studies involving in-service education in mathematics for elementary school teachers indicate that improvement in background knowledge is a result of in-service work (Brown, 1965; Green, 1968; Harper, 1964; Lindsay, 1966; Todd, 1966; Weaver, 1966; Whitman, 1966). Huettig and Newell (1966) found that teachers had a better attitude toward curricular change in mathematics after taking courses in modern mathematics content. DeVault, Houston, and Boyd (1963) recommended that procedures for individualizing instruction be carefully considered, while Dutton and Hammond (1966) found that courses adapted to individual needs were most effective.

The use of computer-assisted instruction (CAI) is one means for providing individualized instruction. The computer allows a varied tutorial sequence to be presented to each learner, with appropriate feedback for his specific responses. Even more important, the learner is actively involved in the learning process. The traditional type of in-service program allows the learner to be passive -- he can sit back and listen or not listen. CAI requires full attention and mental activity.

Several types of computer hardware are available for presenting tutorial CAI courses. The teletype is the simplest of these, but its

flexibility is somewhat limited. Equipment such as the Philco-Ford SAVI is more feasible. IBM 1050 typewriter terminals have been effectively used to present tutorial courses. At present, the IBM 1500 system is widely used. This system is composed of a cathode ray tube (CRT) on which material is presented, and a modified typewriter for use by students in answering. Light pens can be used as an alternate means of response directly on the CRT, and an image projector and audio equipment are available as additional components for presentation of materials. All of these are under computer control. When a student has answered a question, he indicates this to the computer, which then processes his answer and either (a) provides knowledge of a correct response, (b) provides specific feedback for an incorrect answer, or (c) branches the student to appropriate remedial material.

The use of CAI as a vehicle to meet the need to provide individualized instruction on mathematical content to elementary school teachers has been demonstrated by Long and Riedesel (1967). Seventy-eight teachers took the course modmath via CAI, while a comparable control group was given a series of lectures, the typical mode for in-service education courses. Results indicated that the CAI group achieved at least as much as those in the control group: the course taught effectively. One indication of the individualized nature of the CAI program is that the time it took teachers to complete the course ranged from 14 to 47 hours. Reactions of the teachers also indicated their awareness of the individualized experience which the computer program was providing.

The Appalachia Regional Laboratory (1970) used a revised version of the course (elmath) with in-service teachers in three centers in western Virginia and southwestern Pennsylvania. Achievement gain scores were significantly higher, with gains of up to 45 points made on the 80-item pre/post tests. Attitude scores were also more positive after completion of the course. The same course has also been used successfully in two Summer Institutes at The Pennsylvania State University. It seems to be especially effective for the teachers from a disadvantaged area.

Development of the original version of the CAI course modmath was begun six years ago by Dr. C. Alan Riedesel and Dr. Marilyn N. Suydam. Since this version was completed four years ago, it has undergone

continual, extensive revision to improve its effectiveness. A methods component was added to form the present program, elmath. Dr. Cecil R. Trueblood was added to the Riedesel-Suydam author team. The course was developed with a philosophy of the need to guide learning, rather than merely present material about which questions could be asked. Thus, questions are asked to develop the concept. As the student indicates lack of understanding through his incorrect responses, he is branched to sequences lower in the hierarchy which should help to extend his knowledge. The content to be included was planned from CUPM requirements, modified to meet the needs of the in-service audience. The methods material directs teachers to specific strategies, procedures and materials especially useful in teaching elementary school mathematics. An outline of the content of the course is included in Appendix A.

As the course was being used with success with groups of teachers in disadvantaged areas of Appalachia, it seemed apparent that the possibility strongly exists that it can be used effectively with other teachers of the disadvantaged. One such group includes the teachers of Spanish-speaking children, who include a sizable minority in several areas of this country. The possibility of its future use in Puerto Rico or in other Spanish-speaking countries is also being explored.

In order to explore these possibilities and to study the feasibility of its use with Spanish-speaking teachers, a translation of the course was needed. That is the primary purpose of this project: to secure a translated version in Spanish of the elmath CAI course for teaching mathematical content and methods to elementary school teachers.

More than a literal translation is needed, however, since some phrases and some types of experiences may be more appropriately worded. This conceptual adaption formed an integral part of our planning. Experiential background differs, and situations used in the program were changed for this reason. The eventual success of the program may depend on such factors as type of language, manner of presentation, and the whole conceptual organization. The final product of the translation is a program which is conceptually appropriate for the target audience.

Following this phase of the project, other funding will be sought to store the coded materials in the computer. Securing a sound and

conceptually appropriate translation demands concentration of attention. In too many CAI projects, the development of materials phase overlaps with the computer input phase. The pressures created by this on project personnel do not promote the optimal development of the program. By separating these stages, it is the intent of this proposal to investigate the feasibility of this procedure for developing CAI programs.

It should also be decidedly more cost-feasible to make adjustments in previously programmed material should the need arise. Sometimes in the development of CAI programs, the writing of a segment will indicate that an earlier segment should be written differently. Frequently, however, such an adjustment is not made, because of the cost of deleting a segment already in a program and the subsequent insertion of new material. Keeping pressures at a minimal, yet optimal, level should foster the development of a better program.

There is an additional factor which is a positive outcome from this type of procedure: the text of the course is available in a version which will facilitate its encoding for several computer systems. While the re-coding for this projecthas been done for input into the IBM 1500 system, the text could be taken directly and coded for presentation on other equipment. That this is important should be evident to anyone familiar with the status of CAI programs at the moment. Programs are being written for each computer, with little thought of eventual use on another system. The costs of program development are too high to let this situation continue without any attempt to bridge the gap between systems. Since computer manufacturers are reluctant to do this, for obvious reasons, it must become part of the task of program developers.

II. PROCEDURES

The procedures which were followed in the project paralleled those which were enumerated in the proposal:

(1) Development of guidelines. Discussion about and formulation of these considered both the mathematical integrity of the program and the need for a conceptual adaptation. Situations, problems, vocabulary, and structure should be appropriate for the potential Spanish-speaking audience.

(2) Translation of the program. The thirteen-chapter CAI course, and the Handbook, tests, and attitude scale which accompany it, were translated. Students from Chile, Guatamala, Panama, Puerto Rico, and Spain were among those who worked on the translation, under the supervision of Spanish-speaking faculty. They reviewed translated portions to locate errors and made suggestions for improving the presentation. The only real problem met in the project occurred at this reviewing stage, as the perceptions of these persons about the wording most appropriate in their individual Spanish-speaking countries differed. Needless to say, the questions were resolved -- but each new person viewing the translation can be expected to suggest similar changes.

(3) Exploration of potential sites for use. Various agencies, such as the Southwest Educational Development Laboratory and the Ford Foundation, have been contacted. The Ford Foundation sent a team of Chilean educators to review the program for possible implementation in Chile. A copy of part of their report appears in Appendix B.

(4) Preparation for input of CAI course on the system. It was anticipated that some recoding would be done as part of this phase of the project. However, personnel from the CAI Laboratory are developing a computer software package which should eliminate the need to re-code parameters by "hand". Thus the product is now ready for input.

III. RESULTS AND EVALUATION

The primary product of this project is the Spanish translation of elmath: the CAI course and accompanying Handbook, tests, and attitude scale. Because this material is so voluminous (see Table I) and the cost of reproduction would be prohibitive, only a portion of one chapter is attached (see Appendix C). All materials are on file in the Center for Cooperative Research with Schools at The Pennsylvania State University.

No decisions have been reached as to its future use. As the report in Appendix B indicates, it has been recommended for use in Chile; this awaits further action.

It has appeared entirely feasible to translate these materials without inputting the program into the computer system at the same time. To assure this feasibility, a portion of Chapter 1 has been input into the IBM 1500 system in the CAI Laboratory. Chapter 9, which was not previously a part of elmath, was developed by a consultant working off-campus, and is now a part of the translated course.

TABLE I

DATA ON TRANSLATED CAI PROGRAM

	Number of Typewritten Pages
CAI Course	
Content	851
Methods	345
Handbook, Part I	127
Handbook, Part II	181
Achievement Tests	79
Attitude Scale	2

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Appendix A

Content of CAI elmath Course
(Riedesel-Suydam-Trueblood)

Chapter 1: Sets and Early Number Experiences

Content

1. Sets
   a. Elements of sets
   b. Finite and infinite sets
   c. Defined sets
   d. Set notation
   e. Empty set
   f. Universal set
   g. Subsets

2. Set relationships
   a. Equality
   b. Equivalence

3. Set operations
   a. Union
   b. Intersection

4. Complement of a set

Methods

This section focuses attention on why and how sets are presented in early number work. Attention is also directed toward the materials and techniques the teacher should use and the questions she should ask. Levels of pupil performance are considered in terms of types of pupil response.

Chapter 2: Exponents

Content

1. Interpreting exponential notation
   a. Repeated factors
   b. Powers: base and exponent

2. Expressing in exponential form

3. Expanding from exponential form

4. Computation with exponents
   a. Multiplication
   b. Division
   c. Addition and subtraction

5. Zero as an exponent

6. Using expanded notation

Methods

How to teach exponential notation so that pupils see its usefulness is featured. Various pupil-teacher exchanges are presented. Use is made of graph paper and blocks to illustrate exponential forms.

Chapter 3: The Hindu-Arabic System

Content

1. Numerals and word names for numbers

2. Place value
   a. Numerals and names through thousands place
   b. Patterns
      (1) Powers of 10
      (2) Expanded form and standard numerals

   c. Chart: periods and place value through quadrillions
      (1) Reading the numeral
      (2) Completing the chart

Methods

Introducing pupils to the use of place value charts is considered. This is connected with work with the abacus and multi-base arithmetic blocks. The reading of numerals to quadrillions is also considered.

Chapter 4: Other Numeration Systems

Content

1. Introduction to base eight
   a. Symbols: counting
   b. Place value
   c. Changing from base ten to base eight
      (1) Finding powers of the base
      (2) Division by the base

2. Introduction to base five
   a. Changing from base five to base ten
   b. Changing from base ten to base five

3. Characteristics of any numeration system
   a. Number of symbols
   b. Writing the base

4. Introduction to base twelve
   a. Changing from base twelve to base ten
   b. Changing from base ten to base twelve

5. Introduction to base two
   a. Changing from base ten to base two
   b. Changing from base two to base ten

6. Addition in other bases
   a. Base five
   b. Base two

7. Multiplication in base five

Methods

Ways of introducing other numeration systems are presented. Use of materials such as the place value chart is considered, and attention is directed to points at which pupils may have difficulty.

Chapter 5: Addition of Whole Numbers

Content

1. Addition as a binary operation

2. Addition as one of four operations
   a. Relation to subtraction
   b. Relation to multiplication

3. Addition as the union of disjoint sets

4. Counting as the basis for addition: use in problem solving

5. Aids for teaching addition
   a. Abacus
   b. Number line
   c. Cuisenaire rods
   d. Place value frame

6. Properties and principles of addition
   a. Closure
   b. Commutativity
   c. Associativity
   d. Identity element

7. Addition basic facts: use of the table

8. Addition algorithms for multi-digit examples
   a. Use of place value
   b. Use of properties
   c. Regrouping
   d. Expanded notation forms

9. Historical forms for addition
   a. Sandboard method
   b. Scratch method
   c. Front-end addition

10. Checking addition
    a. Excess of nines
    b. Excess of elevens

Methods

For this chapter, the methods component is interwoven with the content. Stress is placed on the use of verbal problems and manipulative materials such as the abacus and Cuisenaire rods.

Chapter 6: Subtraction of Whole Numbers

Content

1. Subtraction on the number line

2. Subtraction as the inverse of addition

3. Terminology
   a. Addend, missing addend, sum
   b. Minuend, subtrahend, difference

4. Subtraction in terms of sets
   a. Complements
   b. Difference between universal set and subset

5. Properties and principles of subtraction
   a. Closure
   b. Commutativity
   c. Associativity
   d. Compensation and renaming

6. Subtraction basic facts: use of the addition table

7. Subtraction algorithms
   a. For basic facts
      (1) Additive method
      (2) Take-away method
   b. For multi-digit examples
      (1) Decomposition
         (a) Additive
         (b) Take-away
      (2) Equal additions
         (a) Additive
         (b) Take-away

8. Checking subtraction
   a. Adding
   b. Excess of nines
   c. Excess of elevens
   d. Complementary method
   e. Scratch method

9. Subtraction in base eight

Methods

Procedures for introducing subtraction to pupils are developed. Also reintroduction using the abacus as a vehicle is presented, and attention is focused on ways of teaching multi-digit subtraction using expanded notation.

Chapter 7: Multiplication of Whole Numbers

Content

1. Multiplication as repeated addition, using the number line

2. Terminology
   a. Multiplier, multiplicand, product
   b. Factors and product

3. Multiplication in terms of sets

4. Arrays and ordered pairs

5. Properties and principles of multiplication
   a. Identity element
   b. Closure
   c. Commutativity
   d. Associativity
   e. Distributivity

6. Multiplication basic facts: use of the table

7. Multiplication algorithms for multi-digit examples
   a. Regrouping
   b. Use of place value

8. Checking multiplication
   a. Use of properties
   b. Excess of nines
   c. Excess of elevens

9. Historical forms for multiplication
   a. Finger reckoning
   b. Lightning method
   c. Scratch method
   d. Lattice method
   e. Duplation methods

10. Modulus multiplication
    a. Mod 2
    b. Mod 7

Methods

Use of arrays in teaching multiplication is developed. Emphasis is placed on providing pupils with varying methods for finding answers to multiplication questions.

Chapter 8: Division of Whole Numbers

Content

1. Relation of division
   a. To multiplication
   b. To subtraction

2. Terminology
   a. Dividend, divisor, quotient
   b. Types: partition and measurement

3. Properties and principles of division
   a. Closure
      (1) Exact division
      (2) Inexact division
   b. Commutativity
   c. Associativity
   d. Right distributivity
   e. Use of zero except as a divisor
   f. Identity element

4. Division algorithms
   a. For basic facts: use of the multiplication table
   b. For multi-digit examples
      (1) Subtracting groups of the divisor
      (2) Use of place value
      (3) Estimation of quotient
         (a) Approximation
         (b) Compensation
         (c) Determining devisibility

5. Historical forms for division
   a. Galley method
   b. A danda method
   c. Division by factors
   d. Excess of nines

6. Division in base four

Methods

Procedures for the diagnosis of pupil difficulties in division are developed. Provision for individual differences is focused on through the study of procedures for estimating the quotient in division.

Chapter 9: Relations and Functions

Content

1. Definition and illustrations of relations
2. Reflexive relations
3. Symmetric relations
4. Transitive relations
5. Equivalence relations
6. Ordered pairs
7. Relations as ordered pairs
8. Relations from one set to another
9. Functions

Chapter 10: Integers

Content

1. Defining the set of integers
   a. Negative signed numbers
   b. Additive inverse

2. Computation with integers

3. Properties and principles of integers
   a. Closure
   b. Commutativity
   c. Associativity
   d. Distributivity
   e. Identity element

4. Order relations of integers

Methods

Three strategies for introducing a lesson are analyzed and compared. The way in which a teacher can use a textbook with other materials is developed.

Chapter 11: Fractions

Content

1. Defining the set of rational numbers

2. Terminology of fractions

3. Uses of fractions
   a. To express parts of a group and parts of a whole
   b. To name a rational number
   c. To indicate division
   d. To express a ratio

4. Characteristics of fractions
   a. Identity element
   b. Equivalence
   c. Cross-products test
   d. Renaming in simplest form
      (1) Prime numbers
      (2) Composite numbers
      (3) Numbers that are relatively prime

5. Order relations of fractions; mixed forms

6. Properties of fractions
   a. Commutativity
   b. Associativity
   c. Distributivity

7. Computation with fractions
   a. Addition
      (1) Like denominators
      (2) Unlike denominators
   b. Finding the L.C.M.
   c. Finding the G.C.D.
   d. Subtraction
      (1) Like denominators
      (2) Unlike denominators
   e. Multiplication
   f. Division
      (1) Common denominator method
      (2) Multiplicative inverse method (inverse)

Methods

Attention is focused on a lesson plan for summarizing the various uses of fractions. The selection of behavioral objectives and analysis of strengths and weaknesses of the plan are included.

Chapter 12: Decimals

Content

1. Place value for decimals

2. Reading and writing decimals

3. Locating decimals on the number line

4. Renaming
   a. Fractions as decimals
   b. Decimals as fractions

5. Terminating decimals

6. Non-terminating decimals
   a. Repeating
   b. Non-repeating

7. Computation with decimals

Methods

Use of a physical world situation to introduce decimals is emphasized with the presentation of a lesson with an odometer. Pupil participation through the use of multiple solutions is reviewed, and non-verbal problems are suggested.

Chapter 13: Ratio and Per Cent

Content

1. Ratio
   a. Expressing ratios
   b. Solving problems with ratios
   c. Using the cross-product method

2. Per cent
   a. Three types of problems
      (1) What is N% of a number?
      (2) What per cent is one number of another number?
      (3) Find the total (100%) when a per cent is known
   b. Five approaches to solving each type of problem
      (1) Decimal
      (2) Ratio
      (3) Unitary-analysis
      (4) Formula
      (5) Equation

Methods

This section is essentially a review and test of material presented in Chapter 10 of the course textbook by Riedesel. When and how ratio and per cent should be developed are emphasized.

Appendix B

Report on the elmath Mathematics Course
(College of Education of The Pennsylvania State University)
by Adriana Campus and Manuel Alarcon, Chile

1. Description of the course. Elmath is an Elementary School Mathematics course designed for the improvement of Elementary School teacher's training. It has 13 chapters in both content and methods. It is a programed course to be taken through an IBM computer with a special instruction equipment IBM 1500 consisting of a TV screen, an image projector and an audio unit.

The 13 content chapters cover essential topics for the basic training of a teacher of the subject: sets, number systems, exponents, operations, properties of operations, functions and a treatment of the rationals and decimals leading to the notion of real numbers.

The content chapters are programed in an exceedingly clear manner, and the way it has been programed gives, by itself, a certain possibility of dialogue between the student-teacher and the person who programed it. The computer very adequately pressures the person working with it, forcing him to make a new effort and offering him help through suggestions and adequate information.

The coexistence of teaching materials such as graphs, tables and practical situations makes the work lively and offers a continuous motivation which maintains conditions favorable for learning.

From the professional point of view, the results obtained by showing the appropriate methods after having mastered the skills is very effective. This broadens the possibilities of each content chapter, giving a very favorable feeling of security to the teacher-student and also an incentive to continue his teaching work. Finally, elmath offers some essential references which complete the information and the learned facts or gives a possibility to do so.

2. Our working procedures. We started with the elmath course. We followed the complete course, all 13 chapters and passed the tests included in it.

We were asked by the College of Education of the University to work in the Spanish translation, which was corrected and reviewed completely by us, and to give this evaluation report.

3. Difficulties. We only had a problem: lack of time, which forced us to prolong our stay in the University. But this problem and the initial ones of language were quickly solved thanks to the valuable cooperation of Dr. Joseph Alessandro, Dr. Bixby and all those helping us.

4. Correction procedure. The complete Spanish translation was corrected directly on the two copies that were given to us, from Chapters 1 to 13 (except for Chapter 9) and the methods part which we corrected directly on the IBM sheets.

This description we offer for the benefit of those who will be working in the programing in Spanish, so that they may find the work easier.

[Note: Specific corrections are cited, but are deleted here.]

5. Applications for Chile. After several meetings for critical appraisal sometimes with the participation of professors of the University we arrived at the following conclusions:

a. The modern curriculum and methods of the course are similar to those introduced in our country, so that with the adaptations being done at the present time, this course could be given in Chile.

i. It would be a most valuable instrument for the "Programa de Colaboracion" or for the "Centro de Perfeccionamiento," to be given to those teachers who need it.

ii. The content part could be used with a similar purpose in secondary school.

iii. It could be a most efficient instrument if administered by each of our 17 Normal Schools throughout the country in order to improve the in-service training of our teachers (about 50,000 in all).

b. The use of computers and instruction laboratories should be carefully considered in its economic aspects due to the high cost of the installations.

c. Even if the use of computers were not possible at this moment, the idea of giving programed instruction courses to in-service teachers seems to us very valuable and perfectly feasible in the form of the appropriate texts; these texts could have the elmath as one of its references.

Conclusions

This appraisal of the practical uses of the elmath course, which we have had the opportunity of studying would render desirable its availability in an educational environment like ours, so spread out and hard to supervise.

In our opinion, elmath could be very fruitful in Chile and any economical effort to have it would be compensated in a short time. Thus, we recommend the study and preparation of projects within the "Programa de Colaboracion para la Ensenanza Normal" (Ministerio de Educacion - Fundacion Ford) with the purpose of bringing about the above mentioned suggestions.

Appendix C

CHAPTER 13: RATIO AND PER CENT

Label	Op Code	Parameter	Text
**ELMAT	013		
ELMAT*E			
1	prr		
INTR13*E			
1	prr		
2	de		
3	ld		:
4	fn		:
5	dt		Este es el Capítulo 13: RAZONES y PORCENTAJES
6	pa		40
M01*E			
1	prr		
2	de		
3	dt		El formato de este capítulo es un tanto diferente al de los capítulos
4	dt		anteriores. En él se le hará una prueba para determinar sus conocimientos en algunas materias. El resultado de la
5	dt		prueba determinará el material adicional que usted recibirá.
6	dti		Presione la barra espaciadora para continuar.
7	epi		
M02*E			
1	prr		
2	de		
3	dt		Exprese la relación "45 millas por 4 galones de gasolina" como una razón de la forma millas/galón. (Use sólo
4	dt		numerales y el símbolo /.)
5	ep		

Label	Op Code	Parameter	Text
6	de		
7	fn		
8	ca		45/4
9	dt		Correcto.
10	pa		20
11	ad		
12	br		M05*E
13	un		
14	dt		Incorrecto.
15	pa		30
16	br		M03*E
M03*E			
1	pr		
2	de		
3	dt		Reconsidere la pregunta: fíjese en que la relación es 45 millas por 4 galones. Trate de expresar denuevo la razón
4	dt		usando el símbolo /.
5	ep		
6	de		
7	fn		
8	ca		45/4
9	dt		Correcto. La razón es 45 es a 4, o bien, 45:4, o bien 45/4.
10	pa		60
11	un		
12	dt		¿Nosotros expresamos la razón como 45/? Conteste denuevo.
13	un		
14	dt		La respuesta es 45/4. Escriba 45/4.
M04*E			
1	prr		
2	de		

Label	Op Code	Parameter	Text
3	dt		Considere: Se tienen naranjas que se venden a $ .55 las 6 naranjas, ¿cuál es la razón naranjas por centavos? (Use
4	dt		sólo numerales y el símbolo /.)
5	ep		
6	de		
7	fn		
8	ca		6/55
9	dt		Correcto.
10	pa		20
11	un		
12	dt		Fíjese: en que estamos comparando naranjas con centavos. Responda denuevo.
13	un		
14	dt		Expresamos la razón como 6/? Pruebe denuevo.
15	un		
16	dt		La respuesta es 6/55. Escriba 6/55.
M05*E			
1	prr		
2	de		
3	dt		¿Qué numeral debe reemplazar a B en (15), de modo que exprese la razón 5 es a 7?
4	ep		
5	de		
6	fn		
7	ca		21
8	cb		2L
9	dt		Correcto.
10	pa		20
11	ad		
12	br		M12*E

Label	Op Code	Parameter	Text
13	un		
14	pa		20
15	dt		Incorrecto.
16	pa		30
17	br		M06*E
M06*E			
1	pr		
2	de		
3	dt		Reconsidere la pregunta: 15 debe tener la misma relación con B como 5 la tiene con _____.
4	ep		
5	de		
6	fn		
7	ca		7
8	dt		Correcto.
9	pa		20
10	un		
11	dt		La respuesta correcta es 7. Escriba 7.
M07*E			
1	pr		
2	de		
3	dt		? x 5 = 15. ¿Qué numeral debería reemplazar el signo de interrogación?
4	ep		
5	de		
6	fn		
7	ca		3
8	dt		Correcto.
9	pa		20
10	un		
11	dt		_ x 5 = 15. Escriba 3.
M08*E			
1	pr		
2	de		

Label	Op Code	Parameter	Text
3	dt		Por lo tanto, para encontrar B, multiplicamos 7 por __.
4	ep		
5	de		
6	fn		
7	ca		3
8	dt		Correcto.
9	pa		20
10	un		
11	dt		7 debe estar multiplicado por 3. Escriba 3.
M09*E			
1	pr		
2	de		
3	dt		¿Entonces qué numeral reemplazaría a B?
4	ep		
5	dc		
6	fn		
7	ca		21
8	dt		Correcto, (15, 21) expresa la razón 5 es a 7.
9	pa		30
10	un		
11	dt		3 x 7 = 21. Escriba 21.
M10*E			
1	pr		
2	de		
3	dt		¿Qué numeral debe reemplazar a B en (20), de modo que exprese la razón 4 es a 9?
4	ep		
5	de		
6	fn		
7	ca		45

Label	Op Code	Parameter	Text
8	dt		Correcto--(20, 45) expresa la razón 4 es a 9.
9	pa		30
10	un		
11	dt		Fíjese en que 20 = 5 x 4. Por lo tanto, B = ? x 9. Responda denuevo.
12	un		
13	dt		20 = 5 x 4, de modo que B = 5 x 9, ó sea ___. Conteste denuevo.
14	un		
15	dt		La respuesta correcta es 45. Escriba 45.
M11*E			
1	pr		
2	de		
3	dt		Fíjese que en el último ejemplo, 4/9 fue multiplicado por 5/5. Cualquier numeral de la forma n/n (n = ?) es otro nombre
4	dt		para el _____ _____ para la multiplicación.
5	ep		
6	de		
7	fn		
8	ca		elemento idéntico
9	dt		Correcto.
10	pa		20
11	aa		
12	ld		
13	fn		
14	fn		
15	dt		Correcto.
16	pa		20
17	un		
18	dt		Cualquier numeral de la forma n/n (n = ?) es otro nombre para el 1, y el 1 es

Label	Op Code	Parameter	Text
19	dt		el elemento idéntico para la multiplicación. Escriba elemento idéntico.
M12*E			
1	prr		
2	de		
3	dt		Una mezcla de concreto 1:2:4 significa: 1 parte de cemento por 2 partes de arena por 4 partes de ripio.
4	dt		¿Cuál es la razón entre la arena y el ripio:
5	ep		
6	de		
7	aa		
8	ld		
9	fn		
10	fn		
11	dt		Correcto. La razón es 2 es a 4, ó 2:4, ò 2/4 (ó 1 es a 2, ó 1:2, ó 1/2).
12	pa		50
13	ad		
14	un		
15	dt		Incorrecto.
16	pa		20
17	ad		
18	br		M13*E
M13*E			
1	pr		
2	de		
3	dt		Para la misma mezcla de concreto 1:2:5, 1 parte de cemento, 2 partes de arena y 4 partes de ripio, ¿indique la razón
4	dt		entre el ripio y el cemento?
5	ep		

Label	Op Code	Parameter	Text
6	de		
7	fn		
8	aa		
9	ld		
10	fn		
11	fn		
12	dt		Correcto. La razón es 4 es a 1 ó 4:1 ó simplemente 4.
13	pa		50
14	ad		
15	ca		4
16	dt		La razón es 4 es a 1, que puede ser expresado simplemente como 4.
17	pa		40
18	ad		
19	un		
20	dt		Incorrecto.
21	pa		20
22	ad		
23	br		M14*E
M14*E			
1	pr		
2	de		
3	dt		En la misma mezcla 1:2:4 de concreto, 1 parte de cemento por 2 partes de arena por 4 partes de ripio, ¿cuál es la razón entre el cemento y el ripio con respecto a la mezcla total?
4	dt		
5	ep		
6	de		
7	aa		
8	ld		
9	fn		
10	fn		
11	dt		Correcto. La razón es 5 es a 7 ó 5/7 ó 5/7.

Label	Op Code	Parameter	Text
12	pa		40
13	ad		
14	aa		
15	ld		
16	fn		
17	fn		
18	dt		Correcto.
19	pa		20
20	ad		
21	br		M20*E
22	un		
23	dt		Incorrecto.
24	pa		20
25	ad		
26	br		M15*E
M15*E			
1	prr		
2	de		
3	br		
4	br		
M16*E			
1	prr		
2	de		
3	dt		Las 3 últimas preguntas estaban relacionadas, y a usted le faltaron por lo menos 2 partes. Consideremos cada parte denuevo.
4	dt		Si la mezcla es 1 parte de cemento por 2 partes de arena y por 4 partes de ripio, la razón entre la arena y el ripio sería 2/4--2 partes por 4 partes. ¿Cuál es la razón entre el ripio y el cemento?
5	dt		
6	ep		
7	de		

Label	Op Code	Parameter	Text
8	aa		
9	ld		
10	fn		
11	fn		
12	dt		Correcto.
13	pa		20
14	un		
15	dt		¿Cuántas partes de ripio se necesitan? 4. ¿Cuántas partes de cemento? 1. Ahora responda denuevo.
16	un		
17	dt		La razón es 4/1. Escriba 4/1.
M17*E			
1	pr		
2	de		
3	dt		1 parte de cemento por 2 partes de arena por 4 partes de ripio. ¿Cuál es la razón entre el cemento y el ripio con respecto a la mezcla total?
4	dt		(Considere: ¿juntas las partes de cemento y ripio? ¿Cuántas partes en la mezcla total?
5	dt		
6	ep		
7	de		
8	aa		
9	ld		
10	fn		
11	fn		
12	dt		Bien.
13	pa		20
14	aa		
15	ld		
16	fn		
17	fn		
18	dt		Correcto.

Label	Op Code	Parameter	Text
19	pa		20
20	un		
21	dt		Usted puede expresar esta razón como 5/7, ó como (4 + 1)/7, ya que comprende 1 parte de cemento más 4 partes de ripio, con respecto a la mezcla total,
22	dt		que es 7 partes. Escriba 5/7.
M18*E			
1	pr		
2	de		
3	dt		Miremos otro ejemplo. El señor Kent le permite a sus niños compartir las ganancias de su jardín en relación con
4	dt		la cantidad de trabajo que cada uno realiza.
5	pa		80
6	dt		
7	dt		El estableció que por cada dólar que Jill reciba, Mike recibirá $3 y Nancy recibirá $5.
8	dt		¿Qué cantidad de dinero recibirá Nancy comparado con lo que recibirá Mike?
9	dt		(Exprese su respuesta en forma de fracción.)
10	ep		
11	de		
12	aa		
13	ld		
14	fn		
15	fn		
16	dt		Correcto.
17	pa		20
18	un		

Label	Op Code	Parameter	Text
19	dt		Nancy recibe $5 por cada $3 que recibe Mike. Esto es 5/? Conteste denuevo.
20	un		
21	dt		La respuesta correcta es 5/3. Escriba 5/3.
M19*E			
1	pr		
2	de		
3	dt		Si Jill recibe $4, ¿cuántos dólares recibe Mike?
4	ep		
5	de		
6	fn		
7	ca		12
8	cb		$12
9	dt		Correcto.
10	pa		20
11	un		
12	dt		La razón entre las ganancias de Jill y las de Mike es 1/3. Por lo tanto la <u>proporción</u> que se necesita para resolver el problema es 1/3 = 4/n. ¿Cuál es el
13	dt		valor para n?
14	un		
15	dt		n = 12. Esto es, Mike recibirá $12. Escriba 12.
M20*E			
1	prr		
2	de		
3	dt		Un impuesto es de $3 por cada $100 de valor declarado. ¿Cuánto impuesto se cobrará por una propiedad tarada en
4	dt		$5600? (Plantee una proporción de la forma a/b = c/d. <u>No</u> resuelva el problema.) Que t represente la cantidad

Label	Op Code	Parameter	Text
5	dt		de impuesto disconocida.
6	ep		
7	de		
8	aa		
9	ld		
10	fn		
11	fn		
12	dt		Correcto.
13	pa		20
14	ad		
15	br		M23*E
16	un		
17	br		M21*E
M21*E			
1	pr		
2	de		
3	dt		Un impuesto es de $3 por cada $100 de valor declarado. ¿Cuánto impuesto se cobrará por una propiedad tasada en
4	dt		$5600? Considere con más cuidado la pregunta. El impuesto puede ser expresado como: 3/?
5	dt		(Escriba el numeral que debe reemplazar al signo de interrogación.)
6	ep		
7	de		
8	fn		
9	ca		100
10	dt		Correcto.
11	pa		40
12	un		
13	dt		La razón es 3/100. Escriba 100.
M22*E			
1	pr		
2	de		

Label	Op Code	Parameter	Text
3	dt		Un impuesto es de $3 por cada $100 de valor declarado. ¿Cuánto impuesto se cobrará por una propiedad tasada en
4	dt		$5600? La cantidad de impuesto, t, con la cantidad de valor declarado. ¿Cómo
5	dt		podría expresarse esto con una razón?
6	ep		
7	de		
8	fn		
9	ca		t/5600
10	dt		Correcto. Entonces usamos el signo igual para establecer la proporción: 3/100 = t/5600.
11	pa		40
12	un		
13	dt		Escriba t/5600.
M23*E			
1	prr		
2	de		
3	dt		Se estima que el mineral acumulado en uno estalagmita es de 2 pulgadas cúbicas por año. A este ritmo, ¿cuántos años
4	dt		demoraría en acumularse un pie cúbico? 1 pie cúbico = 1728 pulgadas cúbicas.
5	dt		Sea n el número de años desconocido. Establezca una proporción de la forma a/b = c/d. No resuelva el problema.
6	ep		
7	de		
8	aa		
9	ld		
10	fn		
11	fn		

Label	Op Code	Parameter	Text
12	dt		Correcto.
13	pa		20
14	ad		
15	br		M27*E
16	aa		
17	ld		
18	fn		
19	fn		
20	dt		Correcto.
21	pa		20
22	ad		
23	br		M27*E
24	un		
25	dt		Incorrecto.
26	pa		20
27	br		M24*E
M24*E			
1	pr		
2	de		
3	dt		Veamos cada parte del problema por separado. El mineral se acumula con un ritmo de 2 pulgadas cúbicas por año.
4	dt		¿Cómo puede expresar esto con una razón?
5	ep		
6	de		
7	aa		
8	ld		
9	fn		
10	fn		
11	dt		Correcto.
12	pa		20
13	un		
14	dt		2 pulgadas cúbicas por año pueden expresarse como 2/1.